



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

of α' from measured values of r' , on the assumption that the value of the constant term is unity, it would be too large by the 0.00000 002 part. And this value substituted in the equation $n' = \sqrt{\frac{M}{a'^3}}$, would give n' too small by the 0.00000 003 part, or n' would be too small by $0''.03895$; or the error in the mean longitude of the sun would amount to nearly $4''$ in a century, a quantity which could not, in the present state of astronomy, be neglected. However, it is only fair to state that astronomers proceed in a way the reverse of this; that is they observe n' and thence deduce α' , and in this case the term 0.00000 002 is without significance, since the logarithms of the radii vectores in the ephemerides are usually given to 7 decimals only.

A CASE OF SYMBOLIC VS. OPERATIVE EXPANSION.

BY A. S. HATHAWAY, CORNELL UNIV., ITHACA, NEW YORK.

Denote by D^m the *symbolic* expansion of $\left(a_1 \frac{d}{da_1} + a_2 \frac{d}{da_2} + \dots\right)^m$, the general term of which is $A_m \alpha_1^r \alpha_2^s \dots \left(\frac{d}{da_1}\right)^r \left(\frac{d}{da_2}\right)^s \dots$, where $A_m = \frac{m!}{r!s!\dots}$, $r+s+\dots=m$; and by $(D)^m$, the operation D or $\left(a_1 \frac{d}{da_1} + a_2 \frac{d}{da_2} + \dots\right)$ repeated m times, whose general term is

$$A_m \left(\left(a_1 \frac{d}{da_1}\right)\right)^r \left(\left(a_2 \frac{d}{da_2}\right)\right)^s \dots;$$

the *extra* parenthesis here, and in what follows, inclosing a symbol which combines *operatively*. Then $(D)^2$ differs from D^2 in the production of an extra term $\alpha_i \frac{d}{da_i}$ by each portion $(D)\alpha_i \frac{d}{da_i}$ [from $\left(\alpha_i \frac{d}{da_i}\right)\alpha_i \frac{d}{da_i}$] of the complete operation $(D)D$ or $(D)^2$; so that the complete $(D)^2 = D^2 + D$. And in general, $(D)D^{m-1}$ differs from D^m by an extra term

$$(r+s+\dots)A_{m-1}\alpha_1^r\alpha_2^s\dots\left(\frac{d}{da_1}\right)^r\left(\frac{d}{da_2}\right)^s\dots$$

derived from each portion

$$(D)A_{m-1}\alpha_1^r\alpha_2^s\dots\left(\frac{d}{da_1}\right)^r\left(\frac{d}{da_2}\right)^s\dots \text{ of the complete operation}$$

$(D)D^{m-1}$; so that, since $r+s+\dots=m-1$, $(D)D^{m-1} = D^m + (m-1)D^{m-1}$.
 $\therefore D^m = [(D-m+1)]D^{m-1} = [(D-m+1)][(D-m+2)]\dots[(D-1)]D$ or $(D)^m$. (1)

Inversely, to find $(D)^m$ in terms of $D^m, D^{m-1}, \dots D$, we have m equations linear in $(D)^m, (D)^{m-1}, \dots (D)$ (obtained by assuming for m in (1) the values $1, 2, 3, \dots m$) from which, if $(m-k)_i$ = the sum of the products of the natural numbers from 1 to $m-k$ inclusive taken i at a time,

$$(D)^m = \begin{vmatrix} D^m & -(m-1)_1 & (m-1)_2 & \dots & \mp(m-1)_{m-1} \\ D^{m-1} & 1 & (m-2)_1 & \dots & \pm(m-2)_{m-2} \\ D^{m-2} & 0 & 1 & \dots & \mp(m-3)_{m-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ D & 0 & 0 & \dots & 1 \end{vmatrix}$$

By another familiar process applied to the symbols $D^m [= (D)^{m'}]$ and $(D)^m$ we get also

$$(D)^m = \sum \frac{D^i 0^m}{i!} D^i.$$

Examples of (1).—1. To change the independent variables in D^m from

$\alpha_1, \alpha_2, \dots$ to $\theta_1, \theta_2, \dots$, where $\alpha_1 = \varepsilon^{\theta_1}, \alpha_2 = \varepsilon^{\theta_2}, \dots$; so that

$$\alpha_1 \frac{d}{d\alpha_1} = \frac{d}{d\theta_1}, \alpha_2 \frac{d}{d\alpha_2} = \frac{d}{d\theta_2}, \dots \quad \left(\left(\alpha_1 \frac{d}{d\alpha_1} + \alpha_2 \frac{d}{d\alpha_2} + \dots \right) \right)^{m'}$$

becomes by the transformation

$$\left(\frac{d}{d\theta_1} + \frac{d}{d\theta_2} + \dots \right)^{m'}. \quad \therefore \text{by (1)} \quad D^m = \left(\frac{d}{d\theta_1} + \frac{d}{d\theta_2} + \dots \right)^{m'}.$$

For a special case see Todhunter's Dif. Cal. Art. 208.

2. By Euler's theorem concerning a homogeneous function $\varphi(\alpha_1, \alpha_2, \dots)$, of n dimensions, $F((D))\varphi = F(n)\varphi; \therefore (D)^{m'}\varphi = n^{m'}\varphi; \therefore$ by (1)

$$D^m \varphi = n^{m'} \varphi.$$

SOLUTION OF A PROBLEM.

BY PROF. E. W. HYDE, UNIVERSITY OF CINCINNATI.

Problem —To show that $\cos^p \varphi \sin^q \varphi$ can be expanded into a series of *cosines* of multiples of φ when q is *even*, and into a series of *sines* of multiples of φ when q is *odd*.

First, suppose q even and $= 2n$, say. Then

$$\begin{aligned} \cos^p \varphi \sin^{2n} \varphi &= \cos^p \varphi (\sin^2 \varphi)^n = \cos^p \varphi (1 - \cos^2 \varphi)^n \\ &= \cos^p \varphi - n \cos^{p+2} \varphi + \frac{n(n-1)}{2!} \cos^{p+4} \varphi - \&c. \end{aligned}$$

Each term of this series can be expanded into a series of cosines of multiples of φ .